Math Goes to Hollywood or: How I Learned to Stop Worrying and Love the Math

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by

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Abstract

Mathematics doesn't put people in the seats at the movie theater, but recently mathematics can be found more and more in popular films. Though Hollywood has shunned the value of mathematics in the past, a trio of excellent films have recently been made with mathematics as an integral character. This paper examines the math topics discussed in three of those movies: *Good Will Hunting*, *Rushmore*, and *Pi*. The mathematics used in each film is analyzed for validity. Also discussed is how each film's depiction of mathematics might effect society's view of mathematics.

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"Tough guys don't do math. Tough guys fry chicken for a living!" These are the words that high school math teacher Jaime Escalante (played by Edward James Olmos) sarcastically exclaims to his students in the film *Stand and Deliver* (IMDB). He is trying to overcome a common stigma shared by many teenagers that math is not cool. It seems that Hollywood filmmakers would tend to agree. Very rarely is a film made to improve the viewer's opinion of mathematics. In general, filmmakers do not like math because it does not allow a bus to ramp over a 100-foot gap in the overpass or allow a plane to land in one piece on the Las Vegas strip.

The place of mathematics in popular culture has not changed much since Plato envisioned math as pure rationality whose presence in nature transcends humanity. Since that time, artists such as Whitman and Millay have come to relate rationality as being inhuman or lacking of emotion. Playwright Tom Stoppard contrasts rationality and romanticism in his recent play *Arcadia,* with both representing opposite ends of a spectrum (Saul). Until recently, the portrayal of math in film has reflected a common view in popular culture that mathematics is cold and rational to a fault.

In the late nineties, three excellent films were made involving real math concepts that may help change the way our society views mathematics. The films *Good Will Hunting*, *Rushmore*, and *Pi* were all critically acclaimed films that raised the awareness and respect for mathematics in general. In very different ways, all three of these films give the viewer a math lesson alongside a moral lesson.

In the movie *Good Will Hunting,* a complex portrayal of mathematicians shows us that even the most rational of thinkers are human as well. It is a film more about relationships than mathematics, but math plays an excellent supporting character. The main character, Will Hunting (played by Matt Damon), is a uniquely gifted orphan with undiscovered math abilities similar to Ramanujan (a self-taught Indian mathematician who extrapolated theories that had baffled mathematicians for years). While mopping the floors of famed mathematics institute MIT, Will's genius is discovered after he is caught solving math problems that were meant to baffle graduate students for months. These math problems are presented in the form of chalkboard challenges from an esteemed MIT professor, Dr. Lambeau (played by Stellan Skarsgard). Lambeau, a former Fields Medalist, is the playboy mathematician (not an oxymoron). As Will's intellectual mentor, Lambeau has to look externally for help supporting Will emotionally. As Will becomes more in touch with his emotions through his relationships with his psychologist and a girlfriend, it is the mathematicians who are left in emotional shambles by the greatness of Will's ability.

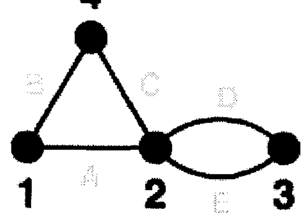
Good Will Hunting does improve the image of mathematics. By portraying mathematicians as emotionally complex and well-rounded characters, we are given a glimpse of mathematicians that is not seen in most movies about math. The film also raises the audience's awareness of greatness in math by peeking inside the inner workings of MIT as well as introducing the Fields Medal as the greatest honor achievable in mathematics.

In *Good Will Hunting* it seems that mathematics is a science that only a few people can ever master (Saul). Many mathematicians are sure to disagree. Math brilliance can be a rare gift but like any gift, its full potential cannot be reached without guidance.

Especially interesting is the portrayal of MIT professor Dr. Lambeau and how his relationships vary. With Will, he is the patient mentor who also does what he can to keep Will out of trouble. This relationship is a foil for the relationship between Lambeau and his graduate student Tom. Though Lambeau is almost never in a scene without his graduate student, Tom does not receive any attention from Lambeau after Will is discovered. Especially fun is Lambeau's relationship with his students. When within the hallowed halls of MIT, Lambeau walks with a swagger. His confidence and flirtatiousness with his students can be seen as they interact over his chalkboard challenges.

Lambeau announces the first chalkboard challenge as an advanced Fourier system, but it turns out to be a less complicated problem in algebraic graph theory. Lambeau boasts that it took his colleagues 2 years to solve that problem, but he hoped that one of his graduate students might solve it in a semester. The winner of the challenge would join the list of former winners that includes "Nobel laureates, Fields medal winners, renowned astrophysicists, and lowly MIT professors." The problem, which is certainly not difficult enough to stump mathematicians for two years, follows:

G is the graph



Find

1) the adjacency matrix A

2) the matrix giving the number of 3 step walks

3) the generating function for walking from point i → j

4) the generating function for walks from points 1 → 3

The first two parts of this problem are simple linear algebra. The adjacency matrix, A, is found by counting the possible walks of length one from each given point. From point one, there are zero possible walks to point one of length one, so the first entry in the matrix is zero. From point one, there is one walk of length one to point two, so the second entry is one. This process is continued until the four by four matrix shown below is obtained.

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

To find the number of three step walks from any given point on the graph, the matrix A is simply cubed:

$$A^3 = \begin{pmatrix} 2 & 7 & 2 & 3 \\ 7 & 2 & 12 & 7 \\ 2 & 12 & 0 & 2 \\ 3 & 7 & 2 & 2 \end{pmatrix}$$

Finding the generating function from point i → j is certainly the most difficult part of this problem. Will's answer is shown as follows:

$$\Gamma^{w}(p_i \to p_j; z) = \sum_{n=0}^{\infty} w_n(i \to j) z^n$$
$$= \frac{\det(I_{ij} - zA_{ij})}{\det(I - zA)}$$

Lambeau obstructs the fourth answer while he checks the validity of the proof, but it is possible to fill in some of the blanks. The generating function for walks from 1 → 3 can be found by simply plugging the numbers into the equation above. The resulting generating function is:

$$\frac{2z^3 + 2z^2}{1 - 7z^2 - 2z^3 + 4z^4}$$

Will follows this with what appears to be a Taylor Polynomial:

$$2z^2 + 2z^3 + 14z^4 + 18z^5 + 94z^6$$

The end of the blackboard cuts off the Taylor polynomial but it is easy to find the coefficients of z to powers higher than six. This is done by taking the number from the 1,3 position of the adjacency matrix to the power of z whose coefficient is being looked for. For example, when A is taken to the sixth power, the number in the 1,3 position is 94. This means there are 94 possible walks of length six from point 1 to 3. The order 12 Taylor Polynomial is shown below.

$$2z^2 + 2z^3 + 14z^4 + 18z^5 + 94z^6 + 146z^7 + 638z^8 + 1138z^9 + 4382z^{10} + 8568z^{11} + 30398z^{12}$$

The second chalkboard challenge, supposed to be more difficult than the first, asks for the number of spanning trees in a complete graph. Without showing any work, Will correctly answers:

$$n^{n-2}$$

The second part of the problem asks, "Draw all the homeomorphically irreducible trees with n = 10." The result is a simple set of eight trees that are more aesthetically pleasing than they are difficult to construct. The proof to this chalkboard problem written by Frank Harary and Geert Prins can be found in the book Acta Math (1959).

After solving these two math problems, Dr. Lambeau tries to help Will find a job using his amazing math skills. Will blows off interviews with the NSA and a fictitious firm called Holden & Macneil. Maybe he should have considered being a movie math consultant. All of the visible math in *Good Will Hunting* was written by Daniel Kleitman, a mathematician at MIT, or Patrick O'Donnell, a physicist at the University of Toronto.

Kleitman was the first mathematician that the filmmakers approached. They wanted to hear a mathematician speak about mathematics so the dialogue in the film would be realistic. Kleitman spoke and worked problems in combinatorics and graph theory, his specific field of study. He made suggestions as to an important unsolved problem that Will could solve, "P=NP". Eventually the decision was made to keep the movie somewhat vague in its discussion of math topics (Kleitman).

O'Donnell not only did the actual handwriting on the chalkboard, but he also found the 2nd chalkboard proof which involved a combination of particle physics and graph theory. After sifting through math books looking for a graffiti-like proof, he finally discovered the graph theory problem by Frank Harary and Deert. O'Donnell also saved the script from the potential goof of making Professor Lambeau a Nobel Prize winner.

Instead, he pointed out that the Fields Medal is the equivalent to winning the Nobel Prize in the field of mathematics (Saul).

The biggest perk for both O'Donnell and Kleitman was the chance to be extras in the film. Kleitman can be seen passing by the window of the deli in Harvard Square where the hero and the heroine have their first kiss. In fact, he passes the window going one direction, and makes a curtain call by passing by in the opposite direction before the end of the scene. O'Donnell appears as a barfly named Marty where he speaks his first and probably last movie line ever, "No shit! You didn't say that." A mathematician, John Mighton, also plays the role of Lambeau's assistant, Tom. Mighton is currently finishing his doctorate at the University of Toronto (Green).

There are other chalkboard proofs shown during *Good Will Hunting* but they were intended to be a backdrop, not a central plot point. Parseval's theorem (mispronounced in the film), chromatic polynomials, and linear algebra provide a wallpaper of chalkboard math that heightens the sense of setting more than it does the plot . Analytically, the math addressed in *Good Will Hunting* is more difficult than any other math related major film to date. The math is still not as complex as the movie claims it to be.

The movie *Rushmore* also overstates the claims of movie math complexity, specifically "The Hardest Geometry Problem in the World." *Rushmore* is a movie about another prodigy of sorts, Max Fischer (played by Jason Schwartzman). Max is considered to be one of the worst students at Rushmore prep academy. What he lacks in book smarts, he makes up for in ambition as he is the president of the bee-keeping club, kite flying society, and many other clubs as well as the writer and director of his own school plays.

As the film opens, fifteen year-old Max daydreams about his geometry class. Max is challenged to solve an extra credit problem, which the teacher put up as a joke. The teacher claims it is "probably the hardest geometry equation in the world." Since the teacher had never seen anyone get it right, including his mentor Dr. Leaky at MIT, he promises that no one in the class will ever have to open a math book again if he can solve it. While calmly drinking his coffee, Max writes the proof to the area of an ellipse equaling "πab" in calligraphy. After his teacher deems the proof correct, Max is lifted onto the shoulders of his cheering classmates before waking up from his daydream. The chalkboard proof is written as follows:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2 - \frac{b^2 x^2}{a^2}$$

$$y^2 = \frac{a^2 b^2 - b^2 x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$\sqrt{y^2} = \sqrt{\frac{b^2}{a^2}(a^2 - x^2)}$$

$$|y| = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$\pm y = \frac{b}{a}\sqrt{a^2 - x^2} \quad \Rightarrow \quad \begin{array}{l} y_2 = +\frac{b}{a}\sqrt{a^2 - x^2} \\ y_1 = -\frac{b}{a}\sqrt{a^2 - x^2} \end{array}$$

$$
\begin{aligned}
\therefore\ A_E &= \int_{-a}^{a} \frac{b}{a}\sqrt{a^2 - x^2} - \{-\frac{b}{a}\sqrt{a^2 - x^2}\}dx \\
&= \frac{4b}{a}\int_{\theta=0}^{\theta=\pi/2} \sqrt{a^2 - a^2\sin^2\theta}\ a\cos\theta\, d\theta \\
&= 4ab\int_0^{\pi/2} \frac{1}{2} + \frac{1}{2}\cos 2\theta\, d\theta \\
&= 2ab[\frac{\pi}{2} - 0] + ab\sin 2\theta\,]_0^{\pi/2} \\
&= \pi ab + ab[\sin\pi - \sin\theta] \\
&= \pi ab
\end{aligned}
$$

If you examine the proof closely, Max does make one simple mistake at the very end of his proof by replacing what should be a "0" with a "θ". Otherwise, the proof is very accurate, which is not surprising since writer/director Wes Anderson is very detail oriented. Another example of this is the use of calligraphy on Max's failed geometry test later in the movie. Max's father reacts to the score of "37" on the test by doctoring it into an "87" and saying, "It could have been worse. You almost got the "A." This also shows us that his father's reaction mirrors the disregard for scholarly study shared by Max.

While Rushmore's depiction of a chalkboard proof is pretty accurate, its difficulty is highly overestimated. Any person with a good calculus background could find the proof for the area of an ellipse. Logically it doesn't make sense that a teacher could simply check the proof from a book, even though his mentor, Dr. Leaky, from MIT had never been able to solve it. These faults are very forgivable since the scene is actually the daydream of a 15-year-old boy. That would also account for the reaction of his classmates who will never have to open a math book again for the rest of their lives.

More important than the actual mathematics used in the scene is the characterization of Max's aspirations of glory in mathematics. It is not clear whether Max is motivated more by the glory of solving the unsolvable problem or the glory of keeping his peers from ever having to open a math book again. Either way, delusions of grandeur in mathematics are a rare gem on the silver screen.

Pi is a movie about another Max, Max Cohen (played by Sean Gullette), a troubled math genius with noble mathematical aspirations. Cohen is searching for a definable truth in mathematics guided by these assumptions:

1.) Mathematics is the language of nature.

2.) Everything around us can be represented and understood with numbers.

3.) If you graph the numbers of any system, patterns emerge. Therefore, there are patterns everywhere in nature.

Using these assumptions, Cohen hypothesizes that there is a pattern in the seemingly chaotic stock market as well. Like pi, the pattern behind the stock market is a mystery. In his work to find the pattern behind the stock market, Cohen's computer spits out a 216-digit bug. Initially he believes it to be trash, but he soon finds out that his mentor, Sol (played by Mark Margolis), also found a similar 216-digit number while attempting to find the pattern behind pi. After irrationally throwing away the print out of the bug, the number in his head becomes the target of Wall Street vultures and Hasidic devils.

There is no chalkboard math in *Pi,* instead the characters use the backs of newspapers. The most intriguing visual math scene involves a Hasidic Jew teaching Cohen about Kaballah, or Jewish mysticism. Ancient Hebrews used the alphabet as their numerical as well as their lettering system. Each letter in the Jewish alphabet has a numerical value. For example the Hebrew "A," is equal to one, while "B," is equal to two and so on. In this way, the entire Torah can be converted into a large string of numbers. When these numbers are analyzed, patterns emerge.

For example, the numeric value of the Hebrew word for "father" summed with the value of the Hebrew word for "mother" is equal to the numeric value of the word for "child". Max begins to see deeper meaning behind the simple Kaballah lesson when he recognizes that the values of the words for "Garden of Eden" and "Tree of Knowledge" are 144 and 233 respectively. These two numbers are part of the Fibonacci sequence (F = 1,1,2,3,5,8,13,21,34,55,89,144,233,...). 144 divided into 233 approaches φ (mistakenly referred to as θ), the golden ratio.

The topic of the golden ratio in *Pi* leads to a discussion of Pythagoras, a mathematician and cult leader who discovered the golden spiral in the 5th century B.C. The spiral is derived via the golden rectangle, a unique rectangle that has the golden ratio. When squared, it leaves a smaller rectangle behind, which has the same golden ratio as the previous rectangle. The squaring can continue indefinitely with the same result. No other rectangle has this trait. When you connect a curve through the corners of these concentric rectangles, you have formed the golden spiral (Gullette).

Interestingly, when Cohen writes the golden ratio, he incorrectly writes it as "A:B::A:A+B" where "A" is the length of the square in the golden rectangle, and "A+B" is equal to the length of the golden rectangle. The correct golden ratio is "B:A::A:A+B" (Reinhold).

The allusion to Pythagoras is interesting because he led a cult which was based on math logic. Pythagoreans loved the golden spiral because they found it everywhere in nature: the nautilus shell, ram's horns, milk in coffee, the face of a Sunflower. It is also found in more modern discoveries such as fingerprints, DNA, and the shape of the Milky Way (Gullette).

The Hasidic Jews in *Pi* resemble a cult similar to how we might picture the Pythagoreans. Dating back to Moses, Kaballah is an ancient form of Jewish mysticism used to speculate on the nature of divinity, creation, and the soul. By studying the Torah (the first five books of the bible), one can unlock the secrets of creation. The practice of using mathematics to unlock these secrets is called Gammantria (Gullette).

In *Pi*, the Jewish cult believes that Max's 216-digit number is the name of God and the key to unlocking the secrets of the Torah. An interesting piece of trivia not mentioned in the movie is the fact that the number 216 is equal to 6 times 6 times 6 (IMDB). From the book of revelations in the New Testament of the bible, "If anyone has insight, let him calculate the number of the beast, for it is man's number. His number is 666." The relevance of this fact is in question since the number 666 would have no meaning to a Jewish sect studying the Torah. Cohen's mentor Sol points out that if you look for any number hard enough, you will see it everywhere. "As soon as you discard scientific rigor, you're no longer a mathematician. You're a numerologist."

Sol gave up looking for the pattern behind pi and now spends his time reading books and playing Go. Go is a simple to learn strategy game born in China over 4000 years ago. A game of Go is played by two people with a Go board and stones. A Go board is a grid of nineteen vertical and horizontal lines that form 361 points of intersection. Players alternately put black and white stones on the points, with the strategy of surrounding the most area. The winner is the player who covers the most area of the board at the end of the game (Gullette).

Go is a fitting game for math geniuses such as Max and Sol to play since it has endless permutations.

> "The Ancient Japanese considered the Go board to be a microcosm of the universe. Although when it is empty it appears to be simple and ordered, in fact, the possibilities of gameplay are endless. They say that no two Go games have ever been alike. Just like snowflakes. So, the Go board actually represents an extremely complex and chaotic universe." -Sol

Go is itself a unique math problem. As computers become more powerful, we are able to program computers that will master chess by evaluating every possible outcome of each move. Programming a computer to play Go is more difficult than Chess because one's position in Go is qualitatively more difficult to evaluate than one's position in Chess. Thus, Go programmers must try to program expert knowledge rather than an exhaustive search method. Human qualities such as perception, judgement, and reasoning are not easily programmable in this case. Currently the best Go computers play at a level considered to be an experienced beginner (Gullette).

Staring at a circular Go stone while riding the subway, Cohen contemplates another major math problem often alluded to in *Pi,* the search for the value of pi. The search for pi is considered to be one of the world's oldest mathematical mysteries. As early as 1650 BCE, there is evidence of the calculation of pi. A scribe in Egypt calculated the value of the ratio of a circle's circumference to its diameter as 3.16.

In 1794, it was proven that pi was an irrational number. Since that time people continue to search for patterns in the string of infinite digits of pi. Calculating the digits to millions of decimal places is one way computer manufacturers test for bugs. Recently Intel discovered a chip bug after testing it with the pi calculation. The importance of the 50 millionth decimal place of pi is arguable since memorizing the first few decimal places is sufficient for most calculations. In 1995, a Japanese man recited from memory 42,000 digits of pi for over nine hours (Gullette).

Pi opens with a title scene showing hundreds of decimal places of pi. Ironically, the numbers are incorrect starting in the ninth decimal places (IMDB). This seems to be a good representation of *Pi* as a movie because topologically it contains a lot of interesting math concepts, but it never goes too far into explaining them past a superficial level. Writer/director Darren Aronofsky jokes, "The hardest math problem in the film is forty-one plus three" (Powell).

A movie does not have to explain the concepts that it discusses in order for the plot to be understood. Often, the goals of the protagonist are made clear to the audience without being entirely relevant to the film. The film term for a device used to move the plot without having much meaning is a "MacGuffin." This term was made notorious by the legendary filmmaker Alfred Hitchcock (Saul). In all three of these films, the mathematics underlying the films is considered to be a MacGuffin. In any movie about mathematics, it is almost necessary that the math be a MacGuffin since graphs and equations don't lend themselves to thrill ride action that most movie audiences are seeking. It's not important to understand how a the math works. Though it may not be as difficult as it seems on film, it is nice to know that someone took the trouble to make use of real math concepts.

Only a mathematician would dissect these excellent math movies to a fault. In the classic film *The Wizard of Oz*, the Scarecrow mistakenly recites the Pythagorean Theorem as, "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side," after receiving his diploma at the end of the movie (Reinhold). Does that mistake make *The Wizard of Oz* less of a great movie?

Mistakes are made, accuracy is questionable but in the end, it is the depiction of math in movies that is most important. If a film prompts a person to explore math topics outside of the film, then it has served a useful purpose (Powell). Anyone who is willing to check a chalkboard proof from an already excellent movie was probably a math lover before the movie. For the viewers that are not already math disciples, a movie that improves their opinion or knowledge of math in any way should be considered a good film.

Maybe in the future there will be a way to make onscreen mathematics more interesting. It is not likely that daredevil mathematicians are intriguing to a general audience, though the math behind a stunt is sure to intrigue a few. Instead, by showing math aspirations as a quality, the three films discussed here are sure to inspire a new generation of math lovers who will look for ways to include mathematics in movies in the future.

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